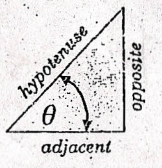
TRIGONOMETRY REVIEW

$F(x)+c$

Definition of the six Trigonometric Functions

Right triangle definitions, where $0<\theta<\frac{\pi}{2}$



$\sin\theta=\frac{opp}{hyp}$ $\quad\csc\theta=\frac{hyp}{opp}$

$\cos\theta=\frac{adj}{hyp}$ $\quad\sec\theta=\frac{hyp}{adj}$

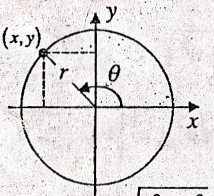
$\tan\theta=\frac{opp}{adj}$ $\quad\cot\theta=\frac{adj}{opp}$

Circular function definition where θ is any angle

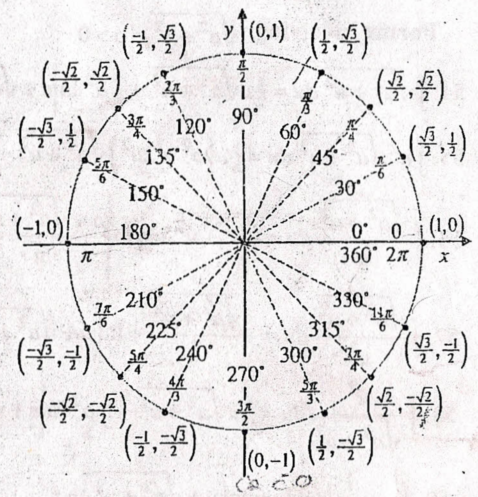
$\sin\theta=\frac{y}{r}$ $\quad\csc\theta=\frac{r}{y}$

$\cos\theta=\frac{x}{r}$ $\quad\sec\theta=\frac{r}{x}$

$\tan\theta=\frac{y}{x}$ $\quad\cot\theta=\frac{x}{y}$



$r=\sqrt{x^2+y^2}$



Reciprocal Identities

$\sin u=\frac{1}{\csc u}$ $\quad\sec u=\frac{1}{\cos u}$ $\quad\tan u=\frac{1}{\cot u}$

$\csc u=\frac{1}{\sin u}$ $\quad\cos u=\frac{1}{\sec u}$ $\quad\cot u=\frac{1}{\tan u}$

Tangent and Cotangent Identities

$\tan u=\frac{\sin u}{\cos u}$ $\quad\cot u=\frac{\cos u}{\sin u}$

Pythagorean Identities

$\sin^2 u+\cos^2 u=1$

$1+\tan^2 u=\sec^2 u$ $\quad 1+\cot^2 u=\csc^2 u$

Co-function Identities

$\sin\left(\frac{\pi}{2}-u\right)=\cos u$ $\quad\cos\left(\frac{\pi}{2}-u\right)=\sin u$

$\csc\left(\frac{\pi}{2}-u\right)=\sec u$ $\quad\tan\left(\frac{\pi}{2}-u\right)=\cot u$

$\sec\left(\frac{\pi}{2}-u\right)=\csc u$ $\quad\cot\left(\frac{\pi}{2}-u\right)=\tan u$

Negative Angle Identities

$\sin(-u)=-\sin u$ $\quad\cos(-u)=\cos u$

$\csc(-u)=-\csc u$ $\quad\tan(-u)=-\tan u$

$\sec(-u)=\sec u$ $\quad\cot(-u)=-\cot u$

Sum and Difference Formulas

$\boxed{\sin(u\pm v)=\sin u\cos v\pm\cos u\sin v}$

$\boxed{\cos(u\pm v)=\cos u\cos v\mp\sin u\sin v}$

$\tan(u\pm v)=\frac{\tan u\pm\tan v}{1\mp\tan u\tan v}$

Double Angle Formulas

$\sin 2u=2\sin u\cos u$

$\cos 2u=\cos^2 u-\sin^2 u=2\cos^2 u-1$

$\cos 2u=1-2\sin^2 u$

$\tan 2u=\frac{2\tan u}{1-\tan^2 u}$

Power Reducing Formulas

$\sin^2 u=\frac{1-\cos 2u}{2}$ $\quad\cos^2 u=\frac{1+\cos 2u}{2}$

$\tan^2 u=\frac{1-\cos 2u}{1+\cos 2u}$

Sum to Product Formulas

$\sin u+\sin v=2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$

$\sin u-\sin v=2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$

$\cos u+\cos v=2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$

$\cos u-\cos v=-2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$

Product to Sum Formulas

$\sin u\sin v=\frac{1}{2}\left[\cos(u-v)-\cos(u+v)\right]$

$\cos u\cos v=\frac{1}{2}\left[\cos(u-v)+\cos(u+v)\right]$

$\sin u\cos v=\frac{1}{2}\left[\sin(u+v)+\sin(u-v)\right]$

$\cos u\sin v=\frac{1}{2}\left[\sin(u+v)-\sin(u-v)\right]$

$\mathrm{sen}(x)\cdot\cos(x)=\frac{\mathrm{sen}\,2x}{2}$

$e^{\theta i}=\cos\theta+i\,\mathrm{sen}\,\theta$

TABLE OF DERIVATIVES

Power of u and Algebraic

1. $D_x(a)=0 \quad\therefore a$ is constant
2. $D_x(x^n)=nx^{n-1}D_xu$
3. $D_x(u\pm v)=D_xu\pm D_xv$
4. $D_x(uv)=u\,D_xv+v\,D_xu$
5. $D_x\left(\frac{u}{v}\right)=\frac{v\,D_xu-u\,D_xv}{v^2}$

Exponential and Logarithmic

6. $D_x(e^u)=e^uD_xu$
7. $D_x(a^u)=a^u\ln a\,D_xu$
8. $D_x(\ln u)=\frac{1}{u}D_xu$
9. $D_x(\log_b u)=\frac{1}{u\ln b}\,D_xu$

7-10

6-9

Trigonometric

10. $D_x(\sin u)=\cos u\,D_xu$
11. $D_x(\cos u)=-\sin u\,D_xu$
12. $D_x(\tan u)=\sec^2 u\,D_xu$
13. $D_x(\cot u)=-\csc^2 u\,D_xu$
14. $D_x(\sec u)=\sec u\tan u\,D_xu$
15. $D_x(\csc u)=-\csc u\cot u\,D_xu$

Inverse Trigonometric

16. $D_x(\sin^{-1}u)=\frac{1}{\sqrt{1-u^2}}D_xu$
17. $D_x(\cos^{-1}u)=\frac{-1}{\sqrt{1-u^2}}D_xu$
18. $D_x(\tan^{-1}u)=\frac{1}{1+u^2}D_xu$
19. $D_x(\cot^{-1}u)=\frac{-1}{1+u^2}D_xu$
20. $D_x(\sec^{-1}u)=\frac{1}{u\sqrt{u^2-1}}D_xu$
21. $D_x(\csc^{-1}u)=\frac{-1}{u\sqrt{u^2-1}}D_xu$

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$|a|<x$

$-x<a<x$

Hyperbolic

22. $D_x(\sinh u)=\cosh u\,D_xu$
23. $D_x(\cosh u)=\sinh u\,D_xu$
24. $D_x(\tanh u)=\mathrm{sech}^2 u\,D_xu$
25. $D_x(\coth u)=-\mathrm{csch}^2 u\,D_xu$
26. $D_x(\mathrm{sech}\,u)=-\mathrm{sech}\,u\tanh u\,D_xu$
27. $D_x(\mathrm{csch}\,u)=-\mathrm{csch}\,u\coth u\,D_xu$

9-12
2-6 > Ma

COLLEGE ALGEBRA REVIEW

Exponent Properties — considering $a, b\neq 0$

1] $a^0=1$

2] $a^{-m}=\frac{1}{a^m}$

3] $\left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}$

4] $a^m\cdot a^n=a^{m+n}$

5] $\frac{1}{a^{-m}}=a^m$

6] $a^m\cdot b^m=(a\cdot b)^m$

7] $\frac{a^m}{a^n}=a^{m-n}=\frac{1}{a^{n-m}}$

8] $\frac{a^{-m}}{b^{-n}}=\frac{b^n}{a^m}$

9] $(a^m)^n=(a^n)^m=a^{mn}$

10] $\left(\frac{a}{b}\right)^{-m}=\left(\frac{b}{a}\right)^m=\frac{b^m}{a^m}$

Properties of Radicals

1] $\sqrt[n]{a}=a^{1/n}$

2] $\sqrt[n]{\sqrt[m]{a}}=\sqrt[nm]{a}$

3] $\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

4] $\sqrt[n]{a\cdot b}=\sqrt[n]{a}\cdot\sqrt[n]{b}$

5] $a^{n/m}=\sqrt[m]{a^n}=\left(\sqrt[m]{a}\right)^n$

6] $a^{n/m}=\left(a^{1/m}\right)^n=\left(a^n\right)^{1/m}$

Factoring Formulas

a] $(x\pm y)^2=x^2\pm 2xy+y^2$

b] $(x\pm y)^3=x^3\pm 3x^2y+3xy^2\pm y^3$

c] $x^2-y^2=(x+y)(x-y)$

d] $x^3\pm y^3=(x\pm y)(x^2\mp xy+y^2)$

Logarithm Properties

a] $\ln e=1$

b] $\ln 1=0$

c] $\ln e^u=u$

d] $e^{\ln u}=u$

e] $\ln(u^v)=v\ln u$

f] $\ln(u\,v)=\ln u+\ln v$

g] $\ln\left(\frac{u}{v}\right)=\ln u-\ln v$

h] $-\ln u=\ln\left(\frac{1}{u}\right)$

$|a|>x$

$a>x \vee a<-x$

TABLE OF INTEGRALS

Basic Forms

1. $\int u\,dv = uv - \int v\,du$
2. $\int u^n\,du = \frac{1}{n+1}u^{n+1} + C \;;\; n \neq -1$
3. $\int \frac{1}{u}\,du = \ln|u| + C$
4. $\int e^u\,du = e^u + C$
5. $\int a^u\,du = \frac{1}{\ln|a|}a^u + C$
6. $\int \sin u\,du = -\cos u + C$
7. $\int \cos u\,du = \sin u + C$
8. $\int \sec^2 u\,du = \tan u + C$
9. $\int \csc^2 u\,du = -\cot u + C$
10. $\int \sec u \tan u\,du = \sec u + C$
11. $\int \csc u \cot u\,du = -\csc u + C$
12. $\int \tan u\,du = \ln|\sec u| + C$ $= -\ln|\cos u| + c$
13. $\int \cot u\,du = \ln|\sin u| + C$
14. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
15. $\int \csc u\,du = \ln|\csc u - \cot u| + C$
16. $\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C$
17. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$
18. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C$
19. $\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$
20. $\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$

Trigonometric Forms

21. $\int \sin^2 u\,du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$
22. $\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$
23. $\int \tan^2 u\,du = \tan u - u + C$
24. $\int \cot^2 u\,du = -\cot u - u + C$
25. $\int \sin^3 u\,du = -\frac{1}{3}\left(2+\sin^2 u\right)\cos u + C$
26. $\int \cos^3 u\,du = \frac{1}{3}\left(2+\cos^2 u\right)\sin u + C$
27. $\int \tan^3 u\,du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C$
28. $\int \cot^3 u\,du = -\frac{1}{2}\cot^2 u - \ln|\sin u| + C$
29. $\int \sec^3 u\,du = \frac{1}{2}\sec u \tan u + \frac{1}{2}\ln|\sec u + \tan u| + C$
30. $\int \csc^3 u\,du = -\frac{1}{2}\csc u \cot u + \frac{1}{2}\ln|\csc u - \cot u| + C$

Inverse Trigonometric Forms

31. $\int \sin^{-1} u\,du = u\sin^{-1} u + \sqrt{1-u^2} + C$
32. $\int \cos^{-1} u\,du = u\cos^{-1} u - \sqrt{1-u^2} + C$
33. $\int \tan^{-1} u\,du = u\tan^{-1} u - \frac{1}{2}\ln\left(1+u^2\right) + C$
34. $\int u\sin^{-1} u\,du = \frac{2u^2-1}{4}\sin^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$
35. $\int u\cos^{-1} u\,du = \frac{2u^2-1}{4}\cos^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$
36. $\int u\tan^{-1} u\,du = \frac{u^2+1}{2}\tan^{-1} u - \frac{1}{2}u + C$

Exponential and Logarithmic Forms

37. $\int ue^{au}\,du = \frac{1}{a^2}(au-1)e^{au} + C$
38. $\int u^n e^{au}\,du = \frac{1}{a}u^n e^{au} - \frac{n}{a}\int u^{n-1}e^{au}\,du$
39. $\int e^{au}\sin bu\,du = \frac{e^{au}}{a^2+b^2}(a\sin bu - b\cos bu) + C$
40. $\int e^{au}\cos bu\,du = \frac{e^{au}}{a^2+b^2}(a\cos bu + b\sin bu) + C$
41. $\int \ln u\,du = u\ln u - u + C$
42. $\int u^n \ln u\,du = \frac{u^{n+1}}{(n+1)^2}\left[(n+1)\ln u - 1\right] + C$
43. $\int \frac{1}{u\ln u}\,du = \ln|\ln u| + C$

Hyperbolic Forms

44. $\int \sinh u\,du = \cosh u + C$
45. $\int \cosh u\,du = \sinh u + C$
46. $\int \tanh u\,du = \ln(\cosh u) + C$
47. $\int \coth u\,du = \ln|\sinh u| + C$
48. $\int \operatorname{sech} u\,du = \tan^{-1}|\sinh u| + C$
49. $\int \operatorname{csch} u\,du = \ln\left|\tanh \frac{1}{2}u\right| + C$
50. $\int \operatorname{sech}^2 u\,du = \tanh u + C$
51. $\int \operatorname{csch}^2 u\,du = -\coth u + C$
52. $\int \operatorname{sech} u \tanh u\,du = -\operatorname{sech} u + C$
53. $\int \operatorname{csch} u \coth u\,du = -\operatorname{csch} u + C$

Forms Involving $\sqrt{a^2+u^2}$; $a>0$

54. $\int \sqrt{a^2+u^2}\,du = \frac{1}{2}u\sqrt{a^2+u^2} + \frac{1}{2}a^2\ln\left|u+\sqrt{a^2+u^2}\right| + C$
55. $\int u^2\sqrt{a^2+u^2}\,du = \frac{1}{8}u\left(a^2+2u^2\right)\sqrt{a^2+u^2} + \frac{1}{8}a^4\ln\left|u+\sqrt{a^2+u^2}\right| + C$
56. $\int \frac{\sqrt{a^2+u^2}}{u}\,du = \sqrt{a^2+u^2} - a\ln\left|\frac{a+\sqrt{a^2+u^2}}{u}\right| + C$
57. $\int \frac{\sqrt{a^2+u^2}}{u^2}\,du = -\frac{\sqrt{a^2+u^2}}{u} + \ln\left(u+\sqrt{a^2+u^2}\right) + C$
58. $\int \frac{u^2}{\sqrt{a^2+u^2}}\,du = \frac{1}{2}u\sqrt{a^2+u^2} - \frac{1}{2}a^2\ln\left|u+\sqrt{a^2+u^2}\right| + C$
59. $\int \frac{1}{\sqrt{a^2+u^2}}\,du = \ln\left(u+\sqrt{a^2+u^2}\right) + C$
60. $\int \frac{1}{u\sqrt{a^2+u^2}}\,du = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2+u^2}}{u}\right| + C$

Forms Involving $\sqrt{a^2-u^2}$; $a>0$

61. $\int \sqrt{a^2-u^2}\,du = \frac{1}{2}u\sqrt{a^2-u^2} + \frac{1}{2}a^2\sin^{-1}\frac{u}{a} + C$
62. $\int u^2\sqrt{a^2-u^2}\,du = \frac{1}{8}u\left(2u^2-a^2\right)\sqrt{a^2-u^2} + \frac{1}{8}a^4\sin^{-1}\frac{u}{a} + C$
63. $\int \frac{\sqrt{a^2-u^2}}{u}\,du = \sqrt{a^2-u^2} - a\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$
64. $\int \frac{\sqrt{a^2-u^2}}{u^2}\,du = -\frac{1}{u}\sqrt{a^2-u^2} - \sin^{-1}\frac{u}{a} + C$
65. $\int \frac{u^2}{\sqrt{a^2-u^2}}\,du = -\frac{1}{2}u\sqrt{a^2-u^2} + \frac{1}{2}a^2\sin^{-1}\frac{u}{a} + C$

Forms Involving $\sqrt{u^2-a^2}$; $a>0$

66. $\int \sqrt{u^2-a^2}\,du = \frac{1}{2}u\sqrt{u^2-a^2} - \frac{1}{2}a^2\ln\left|u+\sqrt{u^2-a^2}\right| + C$
67. $\int u^2\sqrt{u^2-a^2}\,du = \frac{1}{8}u\left(2u^2-a^2\right)\sqrt{u^2-a^2} - \frac{1}{8}a^4\ln\left|u+\sqrt{u^2-a^2}\right| + C$
68. $\int \frac{\sqrt{u^2-a^2}}{u}\,du = \sqrt{u^2-a^2} - a\cos^{-1}\frac{a}{u} + C$
69. $\int \frac{\sqrt{u^2-a^2}}{u^2}\,du = -\frac{\sqrt{u^2-a^2}}{u} + \ln\left(u+\sqrt{u^2-a^2}\right) + C$
70. $\int \frac{1}{\sqrt{u^2-a^2}}\,du = \ln\left|u+\sqrt{u^2-a^2}\right| + C$

$f(x)$ f' f''

i) $f''(x) > 0$ ∪
ii) $f''(x) < 0$ ∩
0 : puntos de inflexión

i) $f'(x) > 0$ crece
ii) $f'(x) < 0$ decrece

1) no derivo
2) indeterminación
3) 2 derivada
4) indeterminación
5) Semáforo de inflexión
6) valor en intervalos y puntos

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